Using Categorical Variables in Regression Analysis

Jonas V. Bilenas
Barclays UK&E RBB
PhilaSUG
June 12, 2013

Outline

- **Quick Review Of Linear Regression Models**
- **What are Categorical Variables?**
- **Coding up Categorical Variables.**
- **Simple Case Studies:**
  1. **Continuous variable and 2-level categorical variable**
  2. **2 Continuous variables and a categorical variable with more than 2 levels.**

Linear Regression Models

$$Y_i = b_0 + b_1 * X_{1_i} + b_2 * X_{2_i} + b_3 * X_{3_i} + \ldots e_i$$

- Y: variable that you would like to predict with a linear model. Also called Dependent Variable.
- X's: Variables you would like to use as predictors of Y in the linear model. Called Independent or Predictor Variables. Can include non linear transformations and interaction terms with other independent variables. Can include categorical (or nominal) information.
- *b*'s: What you are solving for in the model.
- *e*: Error term. Assumption that the errors are normally distributed. Model contains only 1 error term. The b's are solved by minimizing the sum of errors squared ($\sum_{i=1}^{n} e_i^2$).
- *i*: Observation number.
- n: Number of observations.

What are Categorical Variables? Examples?

Statistical Numeric Scales

- NOMINAL
- ORDINAL
- INTERVAL
- RATIO

Coding up Categorical Variables?

Most typical coding is called **Dummy** Coding or **Binary** Coding. The number of Dummy variables you need is 1 less than the number of levels in the categorical level.

Example: Sex: MALE, FEMALE. You have 2 levels, in the regression model you need 1 dummy variable to code up the categories.

LEVEL	SEX
'MALE'	1
'FEMALES'	0

Coding up Categorical Variables? More than 2 Levels

Example: Sex: MALE, FEMALE, OTHER.

You have 3 levels, in the regression model you need 2 dummy variable to code up the categories.

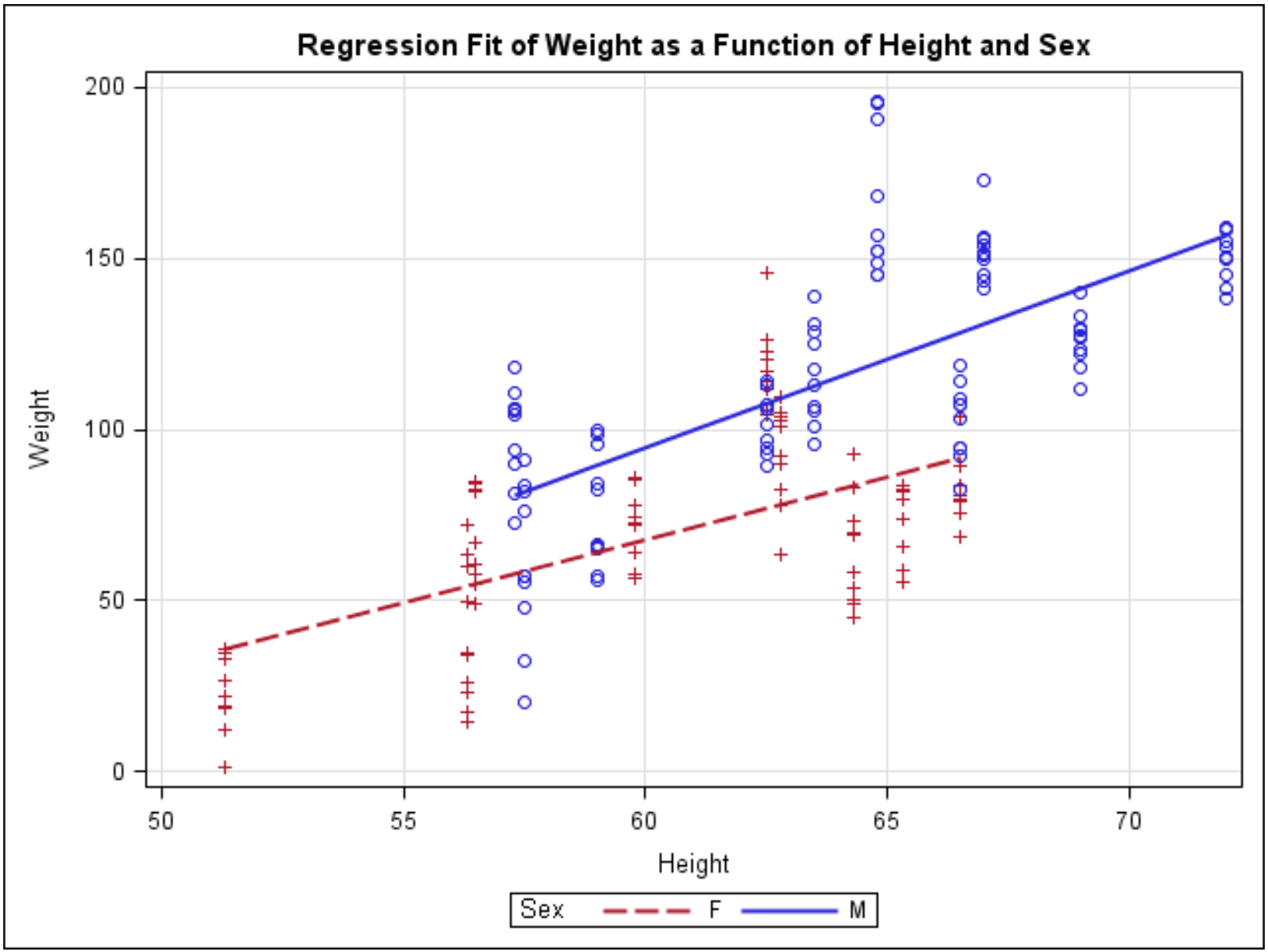
LEVEL	MALE	FEMALE
'MALE'	1	0
'FEMALES'	0	1
OTHER?	0	0

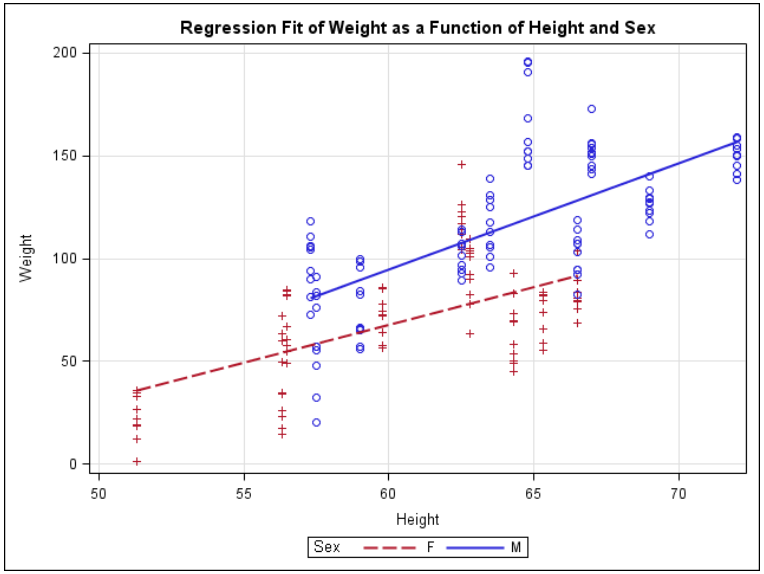
CASE STUDY #1:
Predicting Weight of Children as a function of Height and Sex

Regression with continuous and categorical variables

```
proc sgplot data=class;
  REG x=height y=weight/group=sex;
  xaxis grid;
  yaxis grid;
  title Regression Fit of Weight as a
  Function of Height and Sex;
run;
```

Note: data set CLASS is a modification of SASHELP.CLASS


Regression Fit of Weight as a Function of Height and Sex
Weight
200
150
100
50
0
50
55
60
65
70
Height
Sex
F
M



$$WEIGHT_i = b_0 + b_1 * HEIGHT_i + b_2 * SEX_i + b_3 * HEIGHT_i * SEX_i + \ldots e_i$$

ESTIMATE FOR FEMALES: $WEIGHT_i = b_0 + b_1 * HEIGHT_i$

ESTIMATE FOR MALES: $WEIGHT_i = (b_0 + b_2) + (b_1 + b_3) * HEIGHT_i$

Regression with continuous and categorical variables

```
proc genmod data=class;
   class sex;
   model weight=height | sex
         /dist=NOR wald type3;
run;
```

CLASS: Sets up the coding for you. Be careful that you understand the coding. It may not be DUMMY coding.

TYPE3: Test of the significance of the term after all other terms in the model are added.

Regression with continuous and categorical variables

Class Level Information

Class	Levels	Values
Sex	2	F M

Criteria For Assessing Goodness Of Fit

Criterion	DF	Value	Value/DF
Deviance	186	110150.6871	592.2080
Scaled Deviance	186	190.0000	1.0215
Pearson Chi-Square	186	110150.6871	592.2080
Scaled Pearson X2	186	190.0000	1.0215
Log Likelihood		-874.0435	
Full Log Likelihood		-874.0435	
AIC (smaller is better)		1758.0869	
AICC (smaller is better)		1758.4130	
BIC (smaller is better)		1774.3221	

Regression with continuous and categorical variables

Analysis Of Maximum Likelihood Parameter Estimates

Parameter		DF	Estimate	Standard Error	Wald 95% Confidence Limits		Wald Chi-Square	Pr > ChiSq
Intercept		1	-215.951	32.9369	-280.506	-151.396	42.99	<.0001
Height		1	5.1804	0.5140	4.1730	6.1878	101.58	<.0001
Sex	F	1	62.0376	46.3426	-28.7923	152.8674	1.79	0.1807
Sex	M	0	0.0000	0.0000	0.0000	0.0000	.	.
Height*Sex	F	1	-1.4834	0.7429	-2.9395	-0.0273	3.99	0.0459
Height*Sex	M	0	0.0000	0.0000	0.0000	0.0000	.	.
Scale		1	24.0778	1.2352	21.7746	26.6246		

2 Regression Equations:

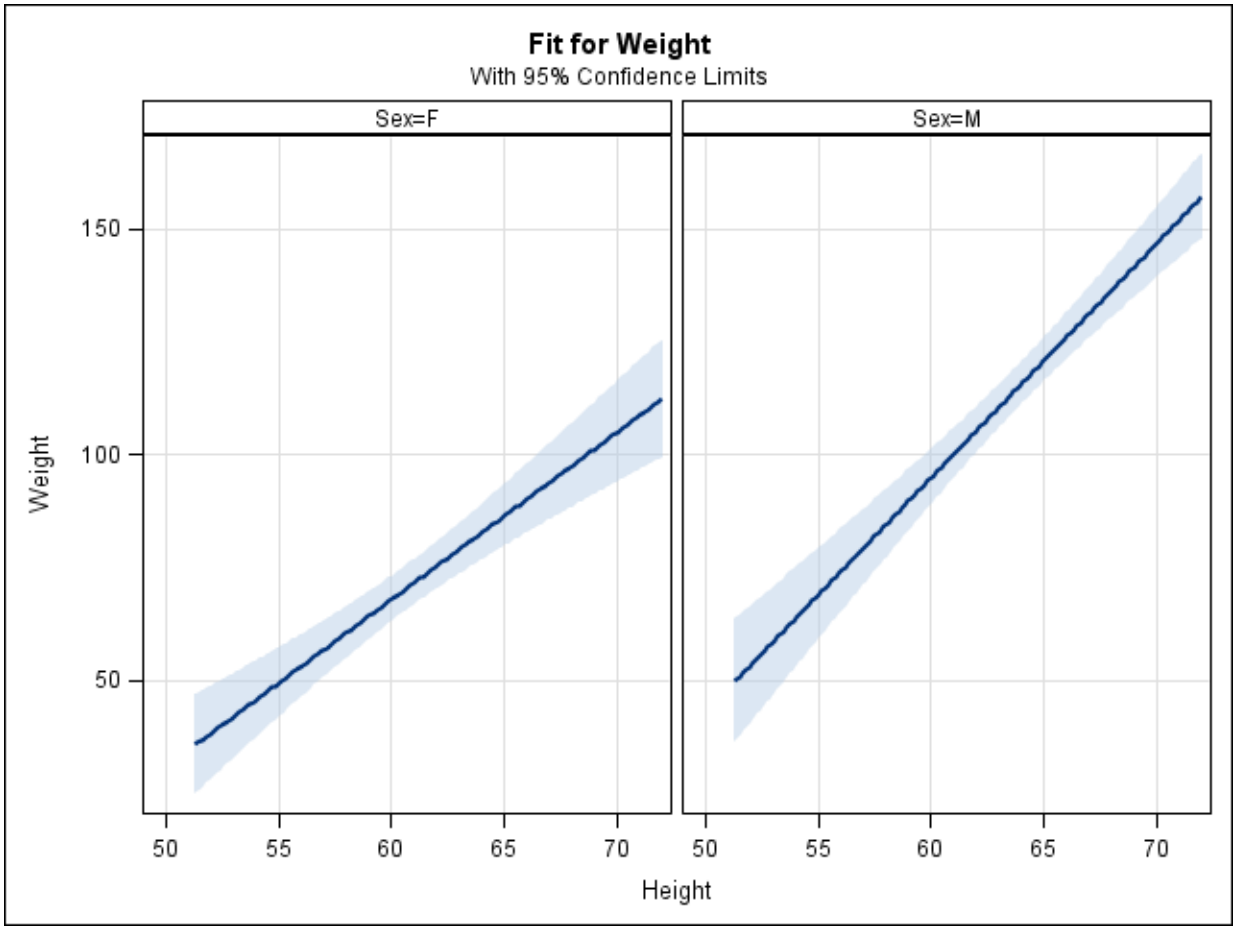
For Males: Weight = -215.951 + 5.1804*Height + e

For Females: Weight = (-215.051+62.0376) +(5.1804 + -1.4834)*Height + e

Effect Plots

```
proc genmod data=class;
   class sex;
   model weight =  Height | Sex
                      / dist=nor
   link=IDENTITY type3 wald;
   effectplot fit(x=Height
   plotby=sex);
run; quit;
```

Regression with continuous and categorical variables

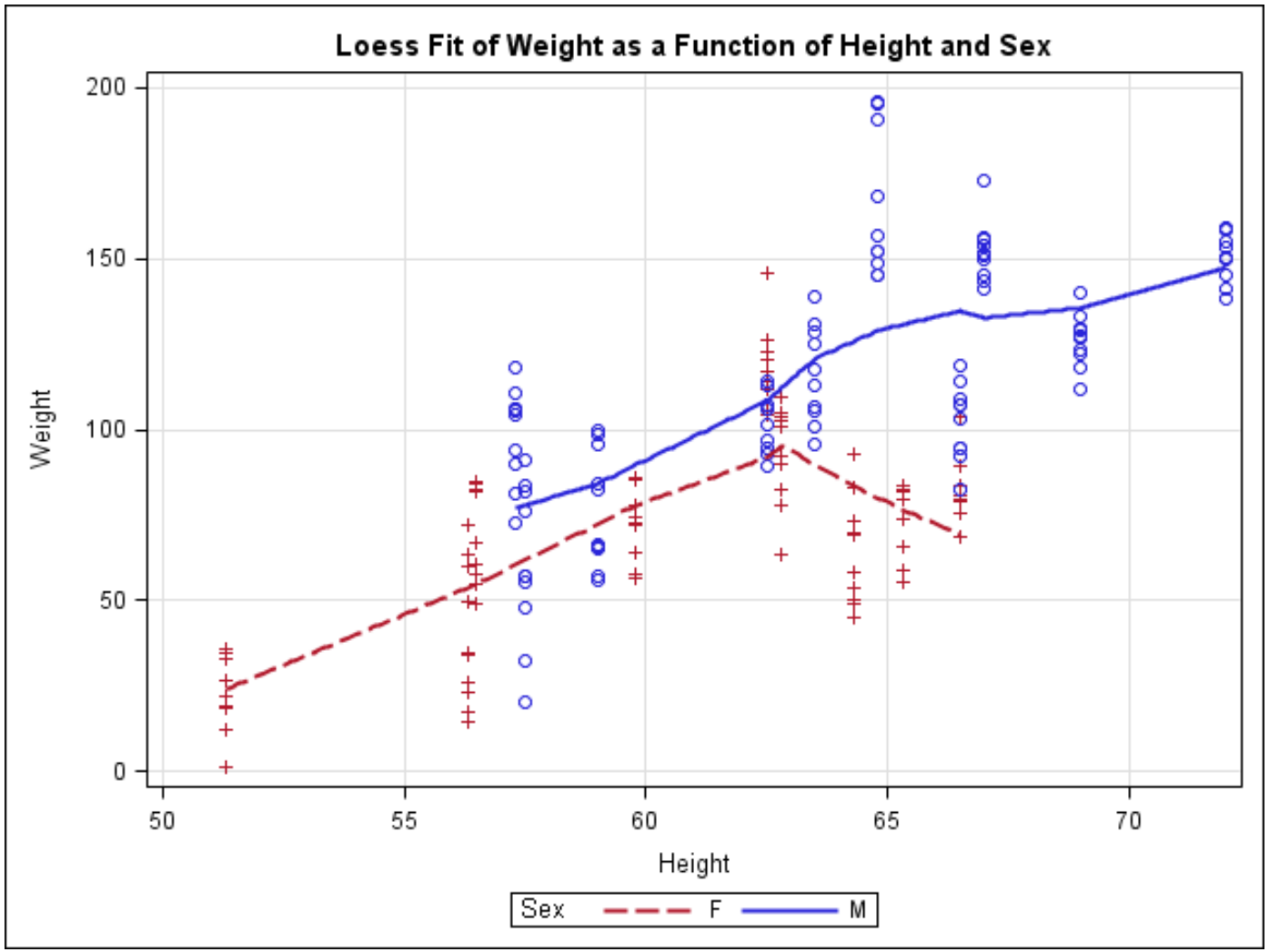


Graphical View

```
proc sgplot data=class;
  loess x=height y=weight/group=sex
                          smooth=0.6;
  xaxis grid;
  yaxis grid;
  title Loess Fit of Weight as a
        Function of Height and Sex;
run;
```

Note: data set CLASS is a modification of SASHELP.CLASS

Graphical View



Original Plots Showed some Curvature

```
proc genmod data=class;
  class sex;
  model weight =  Height | Sex
                  Height*Height | sex
                 / dist=nor link=IDENTITY type3 wald;
  effectplot fit(x=Height plotby=sex);
run; quit;
```

Original Plots Showed some Curvature

Criteria For Assessing Goodness Of Fit

Criterion	DF	Value	Value/DF
Deviance	184	98613.0354	535.9404
Scaled Deviance	184	190.0000	1.0326
Pearson Chi-Square	184	98613.0354	535.9404
Scaled Pearson X2	184	190.0000	1.0326
Log Likelihood		-863.5321	
Full Log Likelihood		-863.5321	
AIC (smaller is better)		1741.0642	
AICC (smaller is better)		1741.6796	
BIC (smaller is better)		1763.7934	

Original Plots Showed some Curvature

Analysis Of Maximum Likelihood Parameter Estimates

Parameter		DF	Estimate	Standard Error	Wald 95% Confidence Limits		Wald Chi-Square	Pr > ChiSq
Intercept		1	-1265.20	440.8444	-2129.24	-401.161	8.24	0.0041
Height		1	38.1508	13.8265	11.0514	65.2503	7.61	0.0058
Sex	F	1	-523.122	597.3555	-1693.92	647.6737	0.77	0.3812
Sex	M	0	0.0000	0.0000	0.0000	0.0000	.	.
Height*Sex	F	1	21.1722	19.4562	-16.9614	59.3057	1.18	0.2765
Height*Sex	M	0	0.0000	0.0000	0.0000	0.0000	.	.
Height*Height		1	-0.2576	0.1080	-0.4692	-0.0460	5.69	0.0170
Height*Height*Sex	F	1	-0.2124	0.1582	-0.5224	0.0976	1.80	0.1793
Height*Height*Sex	M	0	0.0000	0.0000	0.0000	0.0000	.	.

Wald Statistics For Type 3 Analysis

Source	DF	Chi-Square	Pr > ChiSq
Height	1	25.10	<.0001
Sex	1	0.77	0.3812
Height*Sex	1	1.18	0.2765
Height*Height	1	5.69	0.0170
Height*Height*Sex	1	1.80	0.1793

Original Plots Showed some Curvature

```
proc genmod data=class;
  class sex;
  model weight =  Height
                        Height*Height | sex
              / dist=nor link=IDENTITY type3 wald;
  effectplot fit(x=Height plotby=sex);
run; quit;
```

Original Plots Showed some Curvature

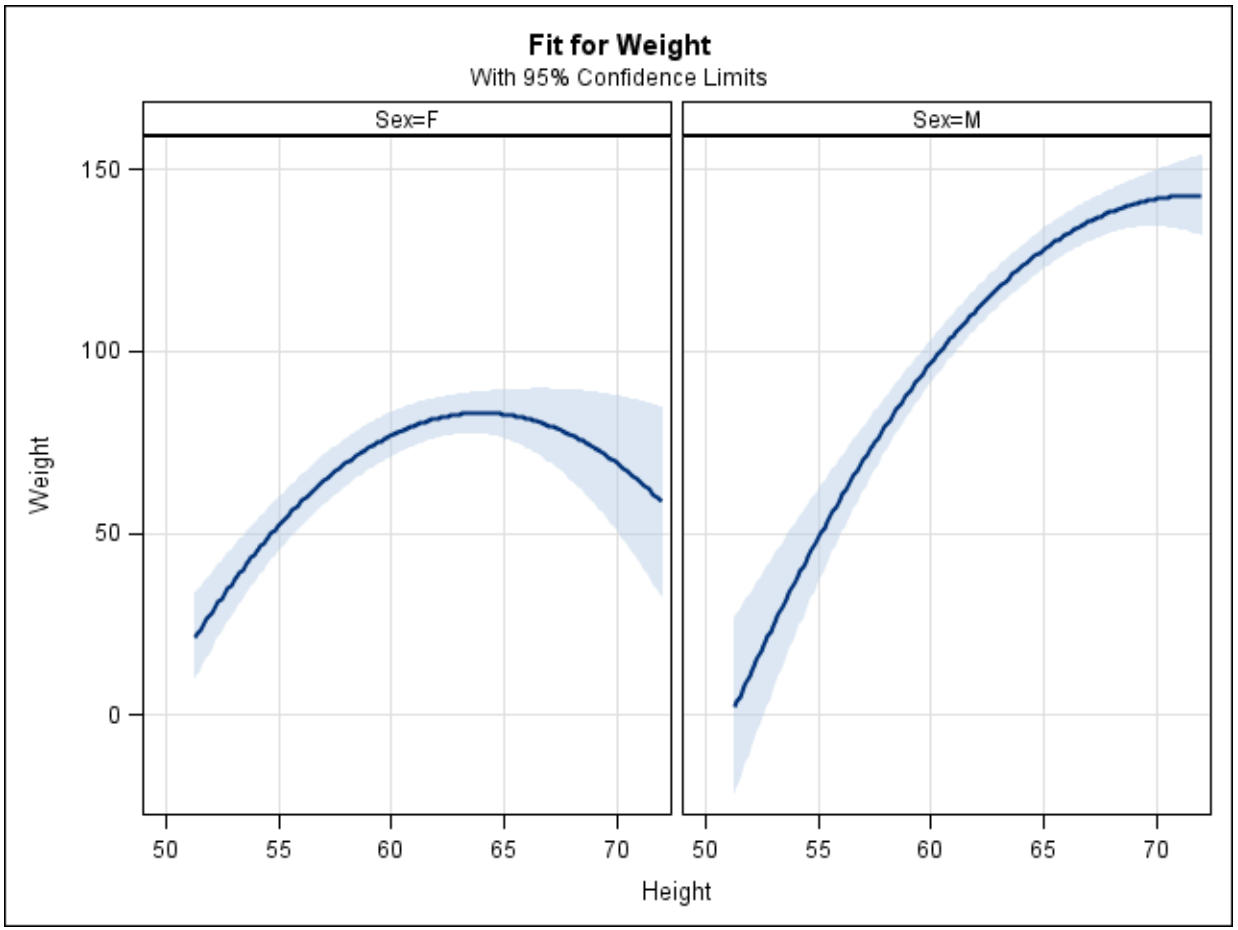
Criteria For Assessing Goodness Of Fit

Criterion	DF	Value	Value/DF
Deviance	185	99227.6359	536.3656
Scaled Deviance	185	190.0000	1.0270
Pearson Chi-Square	185	99227.6359	536.3656
Scaled Pearson X2	185	190.0000	1.0270
Log Likelihood		-864.1224	
Full Log Likelihood		-864.1224	
AIC (smaller is better)		1740.2447	
AICC (smaller is better)		1740.7037	
BIC (smaller is better)		1759.7269	

Analysis Of Maximum Likelihood Parameter Estimates

Parameter		DF	Estimate	Standard Error	Wald 95% Confidence Limits		Wald Chi-Square	Pr > ChiSq
Intercept		1	-1605.90	311.3237	-2216.08	-995.713	26.61	<.0001
Height		1	48.8432	9.7579	29.7180	67.9683	25.06	<.0001
Height*Height		1	-0.3411	0.0762	-0.4905	-0.1916	20.01	<.0001
Sex	F	1	125.9438	32.7730	61.7100	190.1777	14.77	0.0001
Sex	M	0	0.0000	0.0000	0.0000	0.0000	.	.
Height*Height*Sex	F	1	-0.0405	0.0084	-0.0571	-0.0240	23.04	<.0001
Height*Height*Sex	M	0	0.0000	0.0000	0.0000	0.0000	.	.
Scale		1	22.8528	1.1723	20.6668	25.2700		

Original Plots Showed some Curvature



CASE STUDY #2:
Predicting Weight of Children as a function of Height, Age, and Sex.
Sex at 3 levels

What about more than 2 levels. Does it make a difference which level to zero out?

```
proc genmod data=test2;
 class sex;
 model weight = height age sex
          / dist=nor link=IDENTITY type3 wald;
run; quit;
```

```
The GENMOD Procedure

       Model Information

Data Set               WORK.TEST2
Distribution               Normal
Link Function            Identity
Dependent Variable         Weight


Number of Observations Read         958
Number of Observations Used         958


  Class Level Information

Class      Levels    Values

Sex             3    F M X
```

Code for PROC GENMOD using GLM coding:

Analysis Of Maximum Likelihood Parameter Estimates

Parameter		DF	Estimate	Standard Error	Wald 95% Confidence Limits		Wald Chi-Square	Pr > ChiSq
Intercept		1	-140.323	24.1995	-187.754	-92.8933	33.62	<.0001
Height		1	2.2739	0.3129	1.6606	2.8871	52.81	<.0001
Age		1	0.8877	1.0815	-1.2321	3.0074	0.67	0.4118
Sex	F	1	83.4821	15.9398	52.2406	114.7236	27.43	<.0001
Sex	M	1	86.7539	15.2784	56.8088	116.6990	32.24	<.0001
Sex	X	0	0.0000	0.0000	0.0000	0.0000	.	.
Scale		1	29.7687	0.6801	28.4651	31.1319		

NOTE: The scale parameter was estimated by maximum likelihood.

Wald Statistics For Type 3 Analysis

Source	DF	Chi-Square	Pr > ChiSq
Height	1	52.81	<.0001
Age	1	0.67	0.4118
Sex	2	41.45	<.0001

- Read documentation for DEFAULT CLASS CODING!

On Tue, 25 Sep 2007 17:01:01 -0700, Richard <richard.hockey@GMAIL.COM> wrote:
>Does anyone know why SAS didn't choose param=ref as the default? It >seems the obvious choice. >

Maybe because SAS wants you to read the documentation? There is even no consistency across PROCS. Looking at online documentation for the CLASS statement, here are the DEFAULT PARAM= codings depending on the PROC:

PROC	DEFAULT PARAM
SURVEYLOGISTIC	EFFECT
LOGISTIC	EFFECT
TPHREG	REF
GENMOD	GLM

Even if you think you know what you are doing it pays to read the documentation.

Jonas V. Bilenas

http://listserv.uga.edu/cgi-bin/wa?A2=ind0709D&L=sas-l&P=R11996

- EFFECT CODING

SEX	MALE	FEMALE
M	1	0
F	0	1
X	-1	-1

Which level of the categorical variable to make the reference?

I like to make it the level with the highest frequency.

The FREQ Procedure

Sex	Frequency	Percent	Cumulative Frequency	Cumulative Percent
M	500	52.19	500	52.19
F	450	46.97	950	99.16
X	8	0.84	958	100.00

Why? To decrease potential collinearity:

Which level of the categorical variable to make the reference?

To make the highest frequency the reference in GENMOD:

```
proc genmod data=test2;
  class sex/order=freq param=ref ref=first missing;
  model weight = height age sex
                / dist=nor link=IDENTITY type3 wald;
run; quit;
```

Which level of the categorical variable to make the reference in GENMOD?

The GENMOD Procedure

Wald Statistics For Type 3 Analysis

Source	DF	Chi-Square	Pr > ChiSq
Height	1	52.81	<.0001
Age	1	0.67	0.4118
Sex	2	41.45	<.0001

Analysis Of Maximum Likelihood Parameter Estimates

Parameter		DF	Estimate	Standard Error	Wald 95% Confidence Limits		Wald Chi-Square	Pr > ChiSq
Intercept		1	-53.5695	12.3692	-77.8127	-29.3264	18.76	<.0001
Height		1	2.2739	0.3129	1.6606	2.8871	52.81	<.0001
Age		1	0.8877	1.0815	-1.2321	3.0074	0.67	0.4118
Sex	F	1	-3.2718	2.1313	-7.4490	0.9054	2.36	0.1247
Sex	X	1	-86.7539	15.2784	-116.699	-56.8088	32.24	<.0001
Scale		1	29.7687	0.6801	28.4651	31.1319		

Additional Reference mentioned during the presentation:

http://www.nesug.org/proceedings/nesug07/sa/sa07.pdf, P.L. Flom, D.L. Cassell, **Stopping stepwise: Why stepwise and similar selection methods are bad, and what you should use**, NESUG 2007.

SAS® is a registered trademark of SAS Institute.

The contents of this paper are the work of the author and do not necessarily represent the opinions, recommendations, or practices of Barclays or any other company I worked for.

Contact Information:

Email:

jonas.bilenas@barclays.com

jonas@jonasbilenas.com

jonasvb@verizon.net